Data Envelopment Analysis

A way to assess the efficiency of funds of hedge funds.

Greg N. Gregoriou and Joe Zhu

Hedge funds have gained wide acceptance among institutional investors as a way to diversify traditional stock and bond portfolios. Their advantage lies in their low correlations with stocks and bonds and the protection they offer in turbulent markets (Amenc, Bied, and Martellini [2003]).

Hedge fund performance measures using standard market indexes as benchmarks appear questionable, however, as their very nature is different from stock and bond funds. Hedge funds have non-linear returns because of long-short positions, derivatives, and option-like fee contracts, which can result in significant skewness and kurtosis (Fung and Hsieh [1997, 1999]; Liang [2003]; and Agarwal and Naik [2004]).

There is no denying that hedge funds, through leverage, short-selling, and other derivatives strategies, are likely to influence portfolio management by providing downside protection. Yet their inclusion in investor portfolios calls for appraisal methodologies suitable for handling the asymmetrical returns they produce. This is even more important, given that fund of funds manager selection is a precise process for appraising both risk and reward.

There is evidence that using such benchmarks to compare hedge fund and fund of hedge fund (FOF) returns results in low R-squared values (Edwards and Caglayan [2001]; Gregoriou, Rouah, and Sedzro [2002]). We propose a versatile *data envelopment analysis* (DEA) to assess FOF performance.

DEA IN BRIEF

Since data envelopment analysis was introduced in 1978, researchers including us have accepted it as a simple

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and exceptional methodology for modeling processes in performance evaluation (Zhu [2003]; Gregoriou and Zhu [2005]). DEA, a data-oriented approach, uses a linear programming technique to evaluate a set of peer entities called decision-making units (DMUs), whose performance is characterized by multiple measures or indicators. In our case, a DMU refers to a FOFs.

DEA allows for multiple performance measures with different units and generates a best practices or efficient frontier. The multiple performance measures are grouped into inputs and outputs as in production functions. DEA can improve the performance of inefficient FOFs by diminishing input or augmenting output levels.

Exhibit 1 illustrates the fundamental concept of DEA, and how it creates the best practices frontier and establishes benchmarking standards. The vertical axis designates the return as an output, and the horizontal axis designates the standard deviation as an input. Usually, for inputs we prefer lower values (for example, risk measures), and for outputs we prefer higher values (for example, returns).

Using a linear programming technique, DEA identifies a piecewise-linear efficient frontier—the bold solid line as displayed in Exhibit 1. No other observed FOFs have a better risk-return combination than those on the DEA efficient frontier. To enhance the efficiency of FOF D, which is seen as inefficient, its risk must be reduced to D′ on the efficient frontier, or its return must be increased to D″. D′ or D″ is then identified as the benchmark for FOF D.

Exhibit 1 demonstrates that DEA can use either input reduction or output increase for inefficient FOFs to reach the efficient frontier. The efficient frontier is made up of the FOFs where no input reduction and output increase is required. As a result, we have input-oriented DEA models that optimize the inputs (reduce them) but maintain the outputs at their current levels, and output-oriented DEA models that optimize the outputs (increase them) but maintain the inputs at their current levels.

Exhibit 2 shows five FOFs. Each has the same return during one common time period. The two inputs are standard deviation and a proportion of monthly negative return. In this example, FOF4 and FOF5 are relatively inefficient. For example, FOF4 has the same standard deviation as FOF2, but has 15 percentage points more negative monthly returns.

DEA compares the five FOFs in terms of the two inputs and the single output, and identifies FOF1, FOF2, and FOF3 as the best practices units. The efficient frontier consists of the line segments composed by these three efficient FOFs.

DEA identifies T1 on the line segment between FOF1 and FOF2 as the benchmarking standard for the inefficient FOF4. A FOF is rated efficient on the basis of available evidence if and only if the performance of other FOFs does not show that some of its inputs or outputs can be improved without worsening some of its other inputs or outputs.

DEA allows us to rank FOFs in a risk-return framework without using indexes because the FOFs themselves are used as benchmarks. It also permits us to deal with several input and output variables without requiring precise relationships among the multiple performance measures.

An alternative performance evaluation approach like DEA is important because it may enable investors to

EXHIBIT 1

DEA Best Practices Frontier

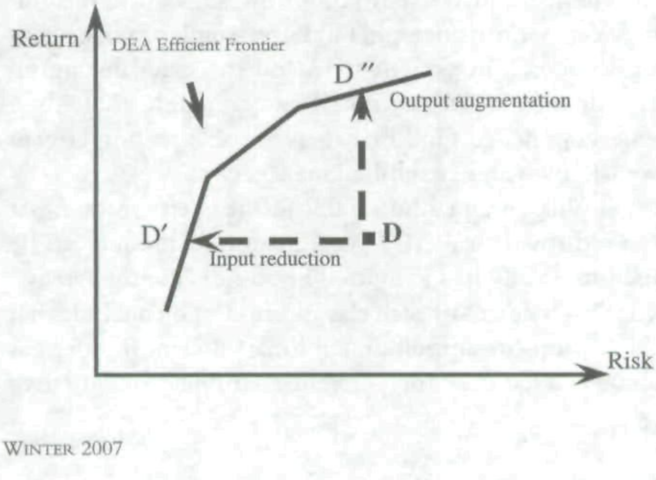
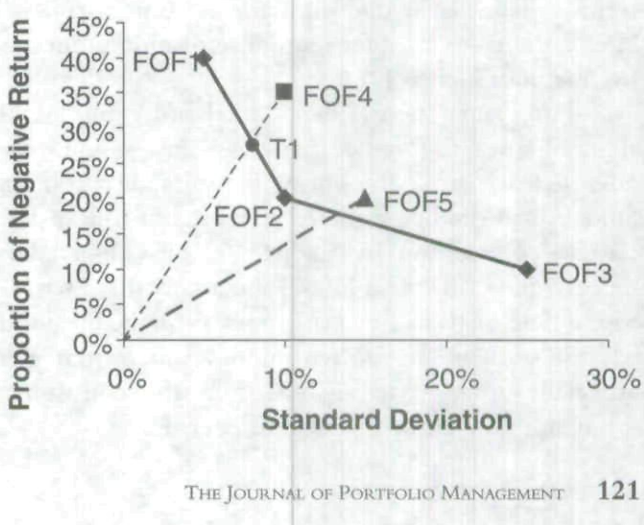


EXHIBIT 2

DEA



pinpoint the reasons behind a FOF's poor performance. For institutional investors and portfolio managers considering FOFs, it is critical that a performance measure provide not only a precise appraisal of the fund's performance, but also an idea of the methods managers use to control risk with respect to certain criteria (variables such as inputs and outputs). DEA can present investors with a useful tool for ranking FOFs by self-appraisal.

Any manager selection process requires assessment of both qualitative and quantitative characteristics. Identifying the best FOF managers through research is surely an art, and alternative performance appraisal techniques such as DEA can provide further insight into fund efficiency.

While investment in a fund of funds can enhance portfolio returns, any enhancement is likely to be limited if the selection process is not followed correctly.

LITERATURE REVIEW

Funds of funds allegedly provide absolute returns; they purport to secure positive returns, irrespective of market conditions. Their special risk exposure and performance characteristics call nevertheless for alternative evaluation techniques. Performance measurement is critical, but research has revealed a tracking error problem for various FOF indexes. FOF indexes are rebalanced and cannot properly reproduce the same composition over an entire performance examination period. Thus persistence could be wrongly estimated.

Amenc, Bied, and Martellini [2003], using multifactor models, partially succeed in predicting hedge fund returns for six of nine hedge fund styles with R-square values ranging from 15.7% to 53.4%. Whatever the ability of typical models to explain hedge fund returns, dynamic trading strategies and skewed returns overall remain a serious matter in hedge fund and FOF performance. Hence, there is a need for more investigation of innovative methods such as DEA.

Fung and Hsieh [1997, 2002a] and Liang [1999] apply Sharpe's factor style analysis to hedge funds, and observe that a small amount of the variability in hedge fund returns can be attributed to those of financial asset classes—contrary to what Sharpe [1992] has observed for mutual funds. L'habitant [2002] also argues that a return-based style analysis performs poorly for hedge funds because of their various dynamic investment strategies, especially since style analysis calls for consistency throughout the entire investigation period.

By application of DEA we aim to alleviate the benchmarking problem in the FOF industry. DEA enables us to rank and evaluate performance and efficiency of offshore and onshore FOFs in a risk-return framework. Diz et al. [2004] take Commodity Trading Advisor (CTA) data from the Barclay Trading Group database and apply basic and cross-efficiency models to examine the efficiency of CTA classifications over the 1997-2001 period. The authors conclude that DEA is a valid technique to identify efficient CTAs, but they use inputs and outputs that are different from ours.

Gregoriou [2006] further addresses trading efficiency of CTAs using DEA. He uses the Center for International Securities and Derivatives Markets (CISDM) CTA database from January 1995 through June 2004 to identify which CTAs produce the highest returns with the fewest trades. Gregoriou, Sedzro, and Zhu [2005] investigate the efficiency of hedge fund classifications over 1997-2001 and 1999-2001. They find DEA is an efficient technique to rank hedge funds but over a short time frame.

Gregoriou and McCarthy's [2005] examination of FOF efficiency does not use any risk-adjusted measures to compare efficiency scores and rankings. Nor do the authors examine any subperiods to identify whether there are differences in the efficiency scores of FOFs.

Gregoriou and Chen [2006] diverge from the traditional DEA models. They use fixed and variable benchmark models to investigate the efficiency of CTA classifications over 1994-2001 using CISDM data. They confirm that the rank-ordering technique is similar to the ranking result produced using the Sharpe ratio.

The traditional Sharpe ratio is considered inappropriate for non-normal returns in comparing efficiency rankings since it underestimates tail risk and overestimates performance. Therefore, we use the modified Sharpe ratio (see Favre and Galeano [2002]).

Using the CISDM CTA database, Darling, Mukherjee, and Wilkens [2004] investigate the relation between performance and fund size, length of a manager's track record, investment style, and strategies during up and down market periods. They conclude that some emerging hedge fund managers are able to outperform well-known and established managers.

Wilkens and Zhu [2003] further demonstrate that the returns-to-scale (RTS) estimation technique can be used to classify hedge funds. The authors use the MAR/CISDM medians of each classification, and conclude that DEA's non-linear multidimensional efficient frontier has an advantage over the conventional linear quantitative methods. In addition Wilkens and Zhu [2001] demonstrate how DEA can be used to benchmark portfolios.

DATA

We use FOF data from the CISDM database to investigate the largest 25 onshore and offshore FOFs (in terms of ending assets under management) during the ten-year period July 1994 through June 2004. We subdivide the periods from July 1, 1999, through June 30, 2004 (a five-year period), and July 1, 2001, through June 30, 2004 (a three-year period).

We choose this period because it includes many of the extreme market events that have occurred since July 1994 (the Tequila crisis of 1994, the Asian currency crisis of 1997, the Russian ruble crisis of 1998, the Nasdaq bubble in 2000, and the September 11, 2001, attacks in the U.S.). We do not consider data before 1994 because an inherent backfill bias renders it unreliable (Fung and Hsieh [2002b]).

Our dataset includes net monthly returns, with management and performance fees deducted. We use the average monthly T-bill rate for each period in computing the modified Sharpe ratio. We use only live FOFs because defunct funds are considered inefficient. When we add defunct FOFs to each DEA model, we find the order ranking remains the same, and the efficiency scores of the current FOFs are not altered.

The inputs and outputs in the framework are important. They must be chosen carefully, because they are the foundation of the analysis. Using other inputs and outputs for DEA models is not incorrect, but must be justified.

To appraise and rank onshore and offshore FOFs, we use three inputs and three outputs for the analysis. The rule of thumb is that the sample size should be approximately twice the number of inputs plus outputs.

An input is simply any resource used by a FOF to produce its outputs. The first input, monthly average standard deviation, is used as a measure of investment risk, because it captures the spread of the returns. A low standard deviation represents a low probability of extraordinary gains or losses; a high standard deviation implies a high probability of gains or losses. The size of the standard deviation can indicate the level of a FOF's risk, and specify which FOFs are more efficient at controlling or minimizing standard deviation.

Next, the monthly downside deviation calculates the average mean return only for periods with a loss, and measures the variation only of the losing periods. This statistic is used to measure the volatility of downside performance and to minimize downside deviation. Thus, we reduce the exposure of FOFs subject to a higher probability of negative returns. Smaller inputs and higher outputs in DEA usually indicate better performance. These inputs are justified because FOFs try to minimize volatility and are concerned mainly with downside volatility.

Finally, the maximum drawdown is used as the third input. Maximum drawdown is the largest drop in percentage terms from the highest peak to the lowest trough before a new peak is attained by the FOF during the period.

Outputs are the result of the processing of inputs. They measure how efficiently an FOF has attained its goals. The first output, monthly percent profitable, refers to the number of positive months divided by the total number of months. This identifies FOFs that are efficient in producing and sustaining the greatest number of positive months throughout the period.

The second output, annualized monthly compounded return, is the constant annual return that results in the identical compound return of the series over the entire period.

Finally, maximum consecutive gain is the number of successive months the FOF has attained positive returns during the period. This output identifies efficient FOFs that are able to maintain a superior level of monthly performance persistence during the period.

The resulting efficiency scores indicate how FOFs rank with respect to their peers.

DEA MODEL

Data envelopment analysis was developed by Charnes, Cooper, and Rhodes [1978] to measure the efficiency of individual decision-making units (DMUs). To present the DEA model, we denote FOFs by $j = \{1, 2, \ldots, n\}$, which uses quantities of i inputs with $i = \{1, 2, \ldots, m\}$ to produce quantities of r outputs with $r = \{1, 2, \ldots, s\}$. We define x_{ij} as the quantity of input i for j used to produce the quantity y_{rj} of output r. Each FOF uses a variable quantity of m different inputs ($i = 1, 2, \ldots, m$) to generate s different outputs ($r = 1, 2, \ldots, s$). FOF j uses amount x_{ij} of output i and generates y_{rj} of output r. We then presume that $x_{ij} \geq 0$, $y_{rj} \geq 0$, and that each FOF has at least one positive input value and one positive output value.

DEA optimization handles the observed vectors of x_j and y_j as given, and selects values of output and input weights for a particular FOF. The efficiency score of a particular FOF can be calculated as follows:*

where:

$$\theta_o^* = \min \theta_o$$

subject to

$$\sum_{j=1}^{n} \lambda_j x_{ij} \le \theta_o x_{io} \quad i = 1, 2, \ldots, m$$

$$\sum_{j=1}^{n} \lambda_j y_{rj} \ge y_{ro} \quad r = 1, 2, \ldots, s \qquad (1)$$

$$\sum_{j=1}^{n} \lambda_j = 1$$

$$\lambda_j \ge 0 \qquad j = 1, 2, \ldots, n$$

m = number of inputs;
s = number of outputs;
n = number of FOFs;
x_{ij} = amount of input i used by FOF j;
x_{io} = amount of input i used by FOF *o under evaluation*;
y_{rj} = amount of output r produced by FOF j; and
y_{ro} = amount of output r produced by FOF *o under evaluation*.

Equation (1) is a linear programming problem. Once solved, the optimal value θ^* indicates the efficiency for a particular FOF under evaluation. If $\theta^* = 1$, the FOF is efficient. If the efficiency score is less than unity, a fund is regarded as inefficient.

In Equation (1), the (unknown) weights λ_j satisfy $\sum_{j=1}^{n} \lambda_j = 1$ This ensures an efficient frontier as shown in Exhibit 1; i.e., a convex combination of efficient FOFs is deemed efficient as well.

Consider the FOFs in Exhibit 2. Applying Equation (1) to FOF 4 yields:

Min θ

subject to

$$5\%\lambda_1 + 10\%\lambda_2 + 25\%\lambda_3 + 10\%\lambda_4 + 15\%\lambda_5 \le 10\%\theta$$
$$40\%\lambda_1 + 20\%\lambda_2 + 10\%\lambda_3 + 35\%\lambda_4 + 20\%\lambda_5 \le 35\%\theta$$
$$\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 \ge 1 \text{ (Same output level)}$$
$$\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 = 1$$
$$\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5 \ge 0$$

and $\theta^* = 0.8$, $\lambda_1^* = 0.4$, $\lambda_2^* = 0.6$, and all other variables = zero, where (*) represents optimal values. This indicates that FOF4 is inefficient when compared to benchmarks FOF1 and FOF2. The non-zero lambda weights indicate that FOF4 is compared to a convex combination of FOF1 and FOF2.

Note that DEA benchmarks are not risk factors, but rather efficient funds as defined in input-output terms where each measure represents risk and return criteria (Wilkens and Zhu [2003]).

RETURNS-TO-SCALE REGION MODEL

A returns-to-scale (RTS) DEA model can be used to rank and classify hedge funds, as shown by Wilkens and Zhu [2003] (see Zhu [2003] for discussion of the RTS model). A FOF manager using leverage may produce a greater-than-proportionate increase in outputs (in particular, returns) relative to fund inputs. In that case, the manager is operating under increasing returns-to-scale. If an increase in FOF inputs results in a less-than-proportionate increase in outputs, the manager is operating under declining returns-to-scale.

We show how DEA under increasing and declining RTS can be used to rank and classify onshore and offshore FOFs. In Exhibit 3, the line segment AB corresponds to increasing RTS (IRS); segment BC corresponds to constant RTS (CRS); and segment CD corresponds to declining RTS (DRS). On the line segment CD, declining RTS appears to the right of point C.

This methodology allows us to allocate a set of FOFs into six RTS regions, as in Exhibit 4. Regions I, II, and III correspond to IRS, CRS, and DRS, respectively. Three additional regions can be identified as well. Region IV refers to input-oriented IRS and output-oriented CRS; Region V refers to input-oriented CRS and

EXHIBIT 3
Returns to Scale

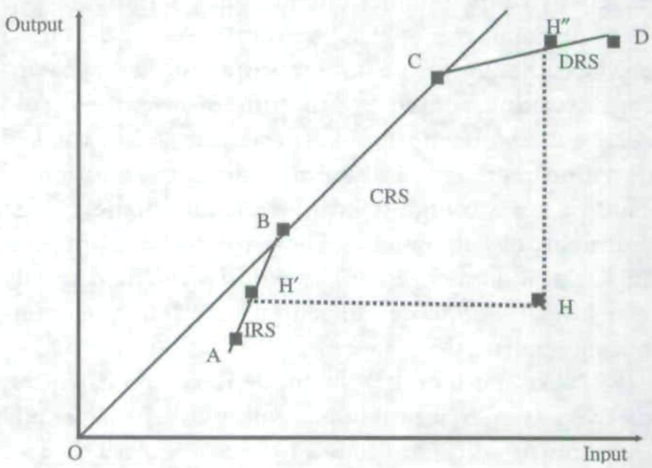
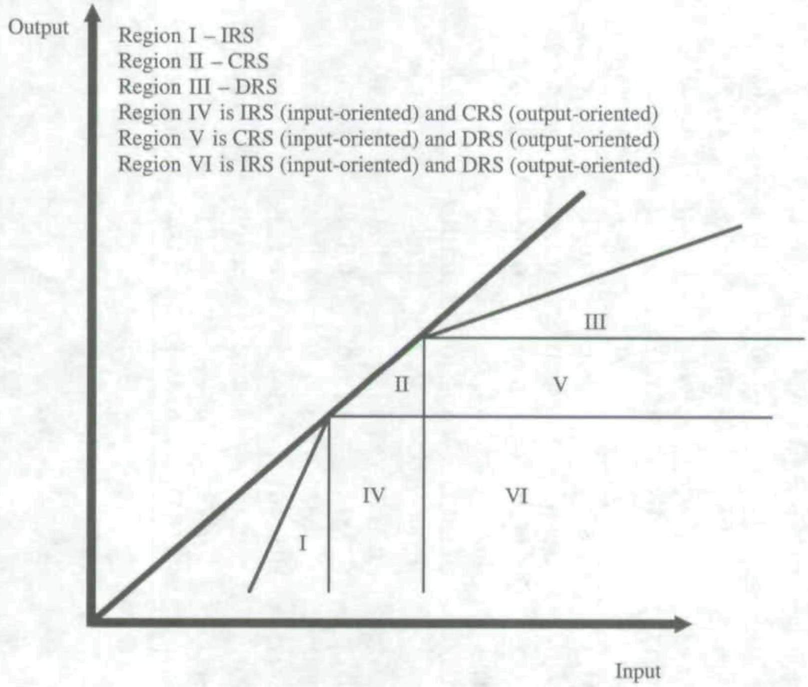


EXHIBIT 4

RTS Regions



output-oriented DRS; and Region VI refers to input-oriented IRS and output-oriented DRS.

A fund of funds manager operating under constant returns-to-scale can scale fund input and outputs linearly, with no increase or reduction in FOF efficiency. Under such an environment, therefore, the increase (or decline) in output is proportional to the increase (or decline) in input. Returns-to-scale, however, need not be constant, but can also be increasing or declining. The efficiency scores from inputs and outputs will be identical under constant RTS. Under varying RTS, however, inputs and outputs are not identical because when inputs are increased outputs can either increase or decline.

Identifying returns-to-scale is important for selecting FOFs because this way we can identify efficiency in using assets under management and other fund resources. For example, constant RTS implies that doubling fund inputs will double outputs. Declining RTS implies that doubling fund inputs will increase outputs by less than double, while increasing RTS will increase outputs by more than double.

The general style classification (GSC) used by Brown and Goetzmann [2003] generates five to eight distinct groups of funds. In that sense, the RTS regions obtained by DEA and presented in Zhu [2003] are similar to the GSC methodology, and to the factor analysis used by Fung and Hsieh [1997]. Unfortunately, the hedge fund classifications provided by database vendors are often not properly defined or may be imprecise, because they are reported by hedge fund managers. Returns-to-scale analysis can group hedge funds into RTS regions, thereby helping to overcome the misclassification that is often inherent in hedge fund databases.

EMPIRICAL RESULTS

Exhibits 5 and 6 show the efficiency scores of offshore and onshore FOFs. A score of 1 implies that a FOF is efficient; a score below 1 would imply it is inefficient. For example, in Exhibit 5, La Fayette Regular Growth Fund, GAM Trading, GEMS Low Volatility Fund, and Global Hedge Fund (CHF) are all efficient during the 1994-2004 period. They represent the best practices frontier, and no other FOF generates the same output level for fewer inputs.

In Exhibit 5, Key Hedge Fund achieves an efficiency score of 0.91174 (or 91.174%), which means the fund is 91.174% efficient using its inputs and outputs. This suggests the fund would need to reduce its inputs by 8.826% to be considered efficient.

When we compare the three-, five-, and ten-year efficiency scores in Exhibit 5, we find there are more efficient funds in the shorter 2001-2004 period than in the other periods. The September 11, 2001, attacks do not seem to have had as much of a negative effect, and FOFs were able to control their inputs. It is more difficult to minimize inputs during very long periods with exposure to many different market environments. Only three offshore funds are efficient in all periods (Lafayette Regular Growth, GAM Trading and Gems Low Volatility).

Many FOFs that attain an efficiency score near 1.0 probably need make only minor corrections to their inputs to be considered efficient. But FOFs with scores well below 80% (for example, Leveraged Capital Holdings, with an efficiency score of 41.13%) are far from efficient (and far from the best practices frontier). FOFs with lowest scores are assumed to be most inefficient. It may be feasible for them to perform input modifications to attain efficiency, but even funds with low scores may attain efficiency

EXHIBIT 5
Efficiency of Offshore Funds of Hedge Funds

FOFs	Efficiency Score July 1994-June 2004	Modified Sharpe Ratio July 1994-June 2004	Efficiency Score July 1999-June 2004	Modified Sharpe Ratio July 1999-June 2004	Efficiency Score July 2001-June 2004	Modified Sharpe Ratio July 2001-June 2004	Ending Millions at June 30, 2004	Compound Return July 1994-June 2004
GAM DIVERSITY FUND	0.61212	0.1103	0.60772	0.3979	0.69870	0.2397	2,963.15	227.02
PERMAL INVESTMENT HOLDINGS NV (A)	0.41686	0.0475	0.16786	0.0310	0.34118	0.0932	2,900.00	177.92
HAUSSMAN HOLDINGS NV	0.47978	0.0696	0.23101	0.0258	0.42029	0.0565	2,700.00	171.89
OLYMPIA STARS TRUST	0.98529	0.2173	0.74482	0.4500	0.67442	0.1928	1,522.35	171.95
LEVERAGED CAPITAL HOLDINGS	0.41134	0.0539	0.19236	0.0061	0.47855	0.0386	1,079.00	182.10
ASIAN CAPITAL HOLDINGS FUND	0.32585	-0.0004	0.17294	0.0493	0.46434	0.1012	722.23	46.74
LA FAYETTE REGULAR GROWTH FUND	1.00000	0.0647	1.00000	0.2814	1.00000	0.3143	630.18	157.62
GAM TRADING FUND	1.00000	0.4278	1.00000	0.4782	1.00000	0.3418	568.08	309.73
GEMS LOW VOLATILITY FUND	1.00000	0.0634	1.00000	0.3618	1.00000	0.3109	427.27	115.70
LA FAYETTE HOLDINGS	0.46122	0.0893	1.00000	0.1502	0.49132	0.1527	416.22	260.80
GLOBAL HEDGE FUND {CHF}	1.00000	0.0357	0.60581	0.1518	0.64732	0.1413	403.27	79.68
GAM EMERGING MARKETS MULTI-FUND	0.26283	0.0218	0.19975	0.0477	1.00000	0.1490	396.65	109.95
GAM MULTI-EUROPE FUND	0.45135	0.0634	0.43319	-0.8560	0.44479	0.0349	315.47	222.67
GREEN WAY INVESTMENTS {EURO}	0.93379	0.0402	1.00000	0.3369	1.00000	0.3252	260.00	95.87
PERMAL EMERGING MKTS HOLDINGS (A)	0.30943	0.0296	0.38832	0.0932	1.00000	0.1666	256.00	122.71
PRIMA CAPITAL FUND	0.48997	0.0409	0.27495	-0.0092	0.41787	0.0021	245.00	130.62
KEY HEDGE FUND	0.91174	0.0890	0.62393	0.1513	0.75230	0.1019	224.57	130.90
SARANAC INVESTORS LTD	0.95181	0.0665	0.43844	0.0964	0.44022	0.0948	221.20	103.10
GAM MULTI-EUROPE FUND {EURO}	0.45775	0.0779	0.47795	-0.3938	0.43155	0.0511	210.94	264.75
GAM TRADING FUND {EURO}	0.71063	0.2913	0.84775	0.3612	1.00000	0.3247	165.70	281.40
OPTIMA FUND LTD	0.62276	0.1520	0.52683	2.7532	0.47855	0.0729	163.70	250.25
TIGER SELECTION HOLDINGS	0.35138	0.0000	0.15648	0.0246	0.39901	0.1239	140.05	47.38
WAFRA STARVEST FUNDS	0.45233	0.0367	0.30041	0.0215	0.58000	0.0191	124.00	114.02
WAFRA STARVEST MULTI-STRATEGY FUND	0.44140	0.0345	0.33733	0.0128	0.50347	0.1080	78.00	124.85
AUDA CLASSIC PLC	0.52061	0.0283	0.24321	-0.0052	0.68354	0.1840	68.81	89.13

Spearman: 0.578 significant at the 0.01 level (July 1994-June 2004).
Spearman: 0.683 significant at the 0.01 level (July 1999- June 2004).
Spearman: 0.784 significant at the 0.01 level (July 2001-June 2004).

EXHIBIT 6
Efficiency of Onshore Funds of Hedge Funds

FOF	Efficiency Score July 1994-June 2004	Modified Sharpe Ratio July 1994-June 2004	Efficiency Score July 1999-June 2004	Modified Sharpe Ratio July 1999-June 2004	Efficiency Score July 2001-June 2004	Modified Sharpe Ratio July 2001-June 2004	Ending Millions at June 30, 2004	Compound Return July 1994-June 2004
MESIROW ALTERNATIVE STRATEGIES FUND	0.56657	0.0923	0.73214	0.3341	0.73973	0.2373	2,207.00	143.91
AURORA	0.62417	0.1154	0.73917	0.2961	0.55554	0.2249	1,404.00	211.41
MERIDIAN HORIZON FUND	1.00000	0.2441	0.79334	-2.5892	0.73543	0.1979	1,145.60	317.78
POINTER (QP)	0.72321	0.3963	0.40399	0.4278	0.51268	0.2528	584.00	239.43
UNIVEST (B)	1.00000	0.4822	0.70303	0.3231	1.00000	0.1972	439.81	220.02
DILLON/FLAHERTY PARTNERS	1.00000	0.2310	1.00000	1.0080	1.00000	1.0029	404.00	165.00
PINE GROVE PARTNERS	0.67702	0.1483	1.00000	0.4539	1.00000	0.3625	367.60	168.18
AUSTIN CAPITAL ALL SEASONS	0.37587	0.1064	0.39590	0.2346	0.53302	0.0297	298.00	161.87
BLUE ROCK CAPITAL FUND	1.00000	0.1004	0.72956	-0.0124	1.00000	-0.0918	224.42	74.65
OPTIMA FUND	0.45230	0.1352	0.33866	-7.4307	0.36475	0.0857	203.40	240.98
PARADIGM MASTER FUND	0.55397	0.1295	0.45179	0.1060	0.45190	0.0623	124.30	120.38
SUMMIT PRIVATE INVESTMENTS	0.34210	0.0911	1.00000	0.1872	1.00000	0.1737	118.08	182.97
ACORN PARTNERS	0.33820	0.0626	0.27703	0.1300	0.45196	0.1384	110.00	122.26
P&A DIVERSIFIED MANAGERS FUND	0.69169	0.1241	0.29143	0.1833	0.45395	0.0089	101.30	365.21
ROSEWOOD ASSOCIATES (D)	0.76336	0.1412	0.79452	0.5564	1.00000	0.3462	100.40	134.26
MILLBURN MCO PARTNERS	0.31720	0.0865	0.26977	0.0713	0.35338	0.0804	97.88	188.14
STELLAR PARTNERS	0.32496	0.0393	0.98961	0.3851	0.90301	0.2325	97.00	214.39
PREFERRED INVESTORS	0.35858	0.1049	0.26126	0.0669	0.67060	0.1488	95.40	191.14
UPSTREAM CAPITAL FUND	0.57857	0.1580	0.64822	0.3591	0.67697	0.3509	86.90	142.04
OXBRIDGE ASSOCIATES	0.42814	0.1083	0.33819	0.1860	0.34500	0.1290	85.00	186.63
P&A MULTI-SECTOR FUND	1.00000	0.1638	1.00000	0.4910	0.50673	0.1017	78.60	443.70
LONG POINT INVESTORS	0.47758	0.0619	0.68808	0.1223	0.67718	0.1130	72.30	264.59
MESIROW ARBITRAGE TRUST	0.63898	0.1143	0.71930	0.3750	0.68830	0.3409	63.40	157.45
REGENCY FUND	0.70423	0.1433	0.68050	0.3857	0.76808	0.2434	61.20	140.10
ACCESS FUND	0.48071	0.1401	0.37061	0.2001	0.46396	0.1297	61.00	143.04

Spearman: 0.764 significant at the 0.01 level (July 1994-June2004).
Spearman: 0.565 significant at the 0.01 level (July 1999-June 2004).
Spearman: 0.554 significant at the 0.01 level (July 2001-June 2004).

if inputs are reduced while outputs (or return) are improved.

DEA essentially gives a realistic representation of each fund's degree of inefficiency, providing a valuable tool for funds that hope to attain 100% efficiency. When efficiency scores are compared against the modified Sharpe ratio using a Spearman's rank correlation coefficient, a strong and positive link is observed in all periods. The robust results in the three-, five-, and ten-year periods validate the precision of the basic DEA model used. We also provide ending assets under management for the 25 FOFs for comparison purposes.

Exhibit 6 shows that the onshore FOFs Meridian Horizon Fund, Univest (B), Dillion/Flaherty Partners, Blue Rock Capital Fund, and P&A Multi-Sector Funds are efficient over the 1994-2004 period. This is only five onshore FOFs on the best practices frontier. Pine Grove Partners, an inefficient FOF with a score of 0.67702, is only 67.702% as efficient as the most efficient FOFs in the analysis. When compared to offshore funds only one onshore fund is efficient in all periods (Dillon/Flaherty Partners).

This does not imply, however, that all efficient FOFs with a score of 1.0 provide the same return during the examination period, and in many cases it does not imply that an efficient fund with a score of 1 will always have the highest compound return. An FOF with an efficiency score of 1.0 returning 20% is considered more volatile than a fund with a score of 1.0 returning 15%, even if both have scores of 1.0.

Exhibit 7 displays the frequency distribution of the basic efficiency scores of the 25 largest offshore and onshore FOFs over all periods. The mean efficiency scores are higher for both offshore and onshore FOFs during 2001–2004, with average basic efficiency scores of 0.6539 and 0.6741, respectively. A possible explanation is that the September 2001 terrorist attacks were not comparable to the magnitute of the August 1998 LTCM/liquidity crisis in Russia resulting in a global meltdown.

In every period, the standard deviation of the efficiency scores is higher for offshore FOFs. A possible explanation could be that offshore FOFs assume more leverage than their onshore counterparts. One interesting finding is that onshore FOFs have higher average mean efficiency scores than offshore FOFs in two of the three periods. This may be because onshore funds are better able to produce higher returns with lower risk than offshore funds.

Exhibit 8 displays the RTS results for four distinct groups of onshore FOFs during the 1994-2004 period. The only FOF in Region I is Blue Rock Capital Fund. This fund produces increasing returns-to-scale (IRS) according to the selected inputs and outputs, implying that an increase in the fund's inputs produces a greater-than-proportional increase in the fund's outputs.

In Region II, two FOFs experience constant returns-to-scale (CRS), which means a proportionate amount of

EXHIBIT 7
Distribution of Efficiency Scores

Efficiency Range	**Offshore FOFs**			**Onshore FOFs**		
	Efficiency			*Efficiency*		
	1994-2004	1999-2004	2001-2004	1994-2004	1999-2004	2001-2004
<0.4	4	11	2	6	8	3
0.4-<0.5	9	3	9	4	2	4
0.5-<0.6	1	1	2	3	0	4
0.6-<0.7	2	3	4	4	3	4
0.7-<0.8	1	1	1	3	7	3
0.8-<0.9	0	1	0	0	0	0
0.9-<1.0	4	0	0	0	1	1
1.0	4	5	7	5	4	6
Sum	25	25	25	25	25	25
Mean	0.6224	0.5188	0.6539	0.6167	0.6246	0.6741
Standard Deviation	0.2635	0.3067	0.2428	0.2368	0.2631	0.2321
Min	0.26	0.16	0.34	0.32	0.26	0.35
Max	1.00	1.00	1.00	1.00	1.00	1.00

EXHIBIT 8

Onshore Funds of Hedge Funds – RTS Region Model

DMU Name	July 1994-June 2004 RTS Region	RTS	July 1999-June 2004 RTS Region	RTS	July 2001-June 2004 RTS Region	RTS
MESIROW ALTERNATIVE STRATEGIES FUND	Region VI	Increasing	Region VI	Increasing	Region VI	Increasing
AURORA	Region III	Decreasing	Region III	Decreasing	Region III	Decreasing
MERIDIAN HORIZON FUND	Region III	Decreasing	Region III	Decreasing	Region VI	Increasing
POINTER (QP)	Region III	Decreasing	Region III	Decreasing	Region VI	Increasing
UNIVEST (B)	Region II	Constant	Region VI	Increasing	Region I	Increasing
DILLON/FLAHERTY PARTNERS	Region II	Constant	Region II	Constant	Region II	Constant
PINE GROVE PARTNERS	Region III	Decreasing	Region III	Decreasing	Region III	Decreasing
AUSTIN CAPITAL ALL SEASONS	Region VI	Increasing	Region VI	Increasing	Region VI	Increasing
BLUE ROCK CAPITAL FUND	Region I	Increasing	Region VI	Increasing	Region I	Increasing
OPTIMA FUND	Region III	Decreasing	Region III	Decreasing	Region VI	Increasing
PARADIGM MASTER FUND	Region VI	Increasing	Region VI	Increasing	Region VI	Increasing
SUMMIT PRIVATE INVESTMENTS	Region III	Decreasing	Region III	Decreasing	Region III	Decreasing
ACORN PARTNERS	Region VI	Increasing	Region VI	Increasing	Region III	Decreasing
P&A DIVERSIFIED MANAGERS FUND	Region III	Decreasing	Region III	Decreasing	Region VI	Increasing
ROSEWOOD ASSOCIATES (D)	Region VI	Increasing	Region VI	Increasing	Region I	Increasing
MILLBURN MCO PARTNERS	Region VI	Increasing	Region VI	Increasing	Region VI	Increasing
STELLAR PARTNERS	Region III	Decreasing	Region III	Decreasing	Region III	Decreasing
PREFERRED INVESTORS	Region VI	Increasing	Region VI	Increasing	Region VI	Increasing
UPSTREAM CAPITAL FUND	Region VI	Increasing	Region VI	Increasing	Region VI	Increasing
OXBRIDGE ASSOCIATES	Region III	Decreasing	Region VI	Increasing	Region VI	Increasing
P&A MULTI-SECTOR FUND	Region III	Decreasing	Region III	Decreasing	Region VI	Increasing
LONG POINT INVESTORS	Region III	Decreasing	Region III	Decreasing	Region III	Decreasing
MESIROW ARBITRAGE TRUST	Region VI	Increasing	Region VI	Increasing	Region VI	Increasing
REGENCY FUND	Region VI	Increasing	Region VI	Increasing	Region VI	Increasing
ACCESS FUND	Region VI	Increasing	Region VI	Increasing	Region VI	Increasing

inputs resulted in a proportionate amount of outputs. In Region III, eleven funds produce declining returns-to-scale (DRS), implying that an increase in the fund's inputs yielded a less-than-proportional increase in the fund's outputs. Finally, in Region VI, eleven FOFs experience increasing returns-to-scale (IRS). The same analysis can be extended for the other two periods.

Exhibit 9 displays the RTS results for four different groups of offshore FOFs during the 1994-2004 period. The only FOF in Region I is the Global Hedge Fund (CHF), yielding increasing returns-to-scale. Region II has two funds producing constant returns-to-scale, while Region III has one fund producing declining returns-to-scale.

Finally, Region VI has the highest number of funds (21), and all experience increasing returns-to-scale (IRS). A large number of funds as identified in Exhibits 8 and 9 experience increasing returns-to-scale. This could suggest that FOFs tend to increase inputs to attain a greater-than-proportional increase in outputs. The opposite can be said for the FOFs during the 1999-2004 period since declining returns-to-scale dominate. Brown, Goetzmann, and Park [2001] suggest that hedge fund managers may have a tendency to increase their risk or leverage during the second half of the year if the fund has not performed well during the first half.

The reason a large number of FOFs attain increasing returns-to-scale in both Exhibits 8 and 9 could be the inherent volatility of the underlying hedge fund strategies in certain FOFs. Under various market conditions, some hedge fund strategies may assume a greater amount of volatility (input), which could be generated from greater leverage or by simply assuming more risky positions. This would generally provide a greater-than proportional increase in returns (output).

EXHIBIT 9

Offshore Funds of Hedge Funds—RTS Region Model

DMU Name	July 1994-June 2004 RTS Region	RTS	July 1999-June 2004 RTS Region	RTS	July 2001-June 2004 RTS Region	RTS
GAM DIVERSITY FUND	Region VI	Increasing	Region III	Decreasing	Region VI	Increasing
PERMAL INVESTMENT HOLDINGS NV (A)	Region VI	Increasing	Region III	Decreasing	Region VI	Increasing
HAUSSMAN HOLDINGS NV	Region VI	Increasing	Region VI	Increasing	Region VI	Increasing
OLYMPIA STARS TRUST	Region VI	Increasing	Region III	Decreasing	Region VI	Increasing
LEVERAGED CAPITAL HOLDINGS	Region VI	Increasing	Region VI	Increasing	Region VI	Increasing
ASIAN CAPITAL HOLDINGS FUND	Region VI	Increasing	Region III	Decreasing	Region III	Decreasing
LA FAYETTE REGULAR GROWTH FUND	Region III	Decreasing	Region III	Decreasing	Region II	Constant
GAM TRADING FUND	Region II	Constant	Region III	Decreasing	Region III	Decreasing
GEMS LOW VOLATILITY FUND	Region II	Constant	Region II	Constant	Region II	Constant
LA FAYETTE HOLDINGS	Region VI	Increasing	Region III	Decreasing	Region VI	Increasing
GLOBAL HEDGE FUND {CHF}	Region I	Increasing	Region VI	Increasing	Region VI	Increasing
GAM EMERGING MARKETS MULTI-FUND	Region VI	Increasing	Region III	Decreasing	Region III	Decreasing
GAM MULTI-EUROPE FUND	Region VI	Increasing	Region III	Decreasing	Region VI	Increasing
GREEN WAY INVESTMENTS {EURO}	Region VI	Increasing	Region III	Decreasing	Region II	Constant
PERMAL EMERGING MKTS HOLDINGS (A)	Region VI	Increasing	Region III	Decreasing	Region III	Decreasing
PRIMA CAPITAL FUND	Region VI	Increasing	Region VI	Increasing	Region VI	Increasing
KEY HEDGE FUND	Region VI	Increasing	Region VI	Increasing	Region III	Decreasing
SARANAC INVESTORS LTD	Region VI	Increasing	Region VI	Increasing	Region VI	Increasing
GAM MULTI-EUROPE FUND {EURO}	Region VI	Increasing	Region III	Decreasing	Region VI	Increasing
GAM TRADING FUND {EURO}	Region VI	Increasing	Region III	Decreasing	Region III	Decreasing
OPTIMA FUND LTD	Region VI	Increasing	Region III	Decreasing	Region VI	Increasing
TIGER SELECTION HOLDINGS	Region VI	Increasing	Region VI	Increasing	Region III	Decreasing
WAFRA STARVEST FUNDS	Region VI	Increasing	Region VI	Increasing	Region VI	Increasing
WAFRA STARVEST MULTI-STRATEGY FUND	Region VI	Increasing	Region VI	Increasing	Region VI	Increasing
AUDA CLASSIC PLC	Region VI	Increasing	Region VI	Increasing	Region III	Decreasing

CONCLUSION

Cautious investors selecting funds of funds must use more than one method to scrutinize returns, volatility, and other statistical properties of a fund. They can use a DEA model to obtain further insight into what drives FOF performance and into how each fund scores in terms of relative efficiency. DEA groups FOFs into specific regions, such as increasing, declining, or constant returns-to-scale (RTS), just as factor analysis and GSC are frequently used to group hedge funds and FOFs.

We show how DEA can be used as an alternative selection tool to help pension funds, institutional investors, and high-net worth individuals to select the most efficient FOFs. DEA is a valuable complement to other risk-adjusted measures because it widens the picture of FOF performance appraisal. Our empirical results support the contention that even when using non-normal returns in a risk-return framework, DEA can provide reliable results. This advantage is important, given that conventional risk measurement techniques can be misleading and fail to identify the better-performing funds.

Future research using different DEA models could investigate managed futures and other hedge fund classifications. It would also be interesting to measure the relative efficiency of various investable hedge fund and managed futures indexes.

ENDNOTES

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* In the DEA literature, this model is called the variable returns-to-scale (VRS) model (see, e.g., Banker, Charnes, and Cooper [1984] and Zhu [2003].)

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